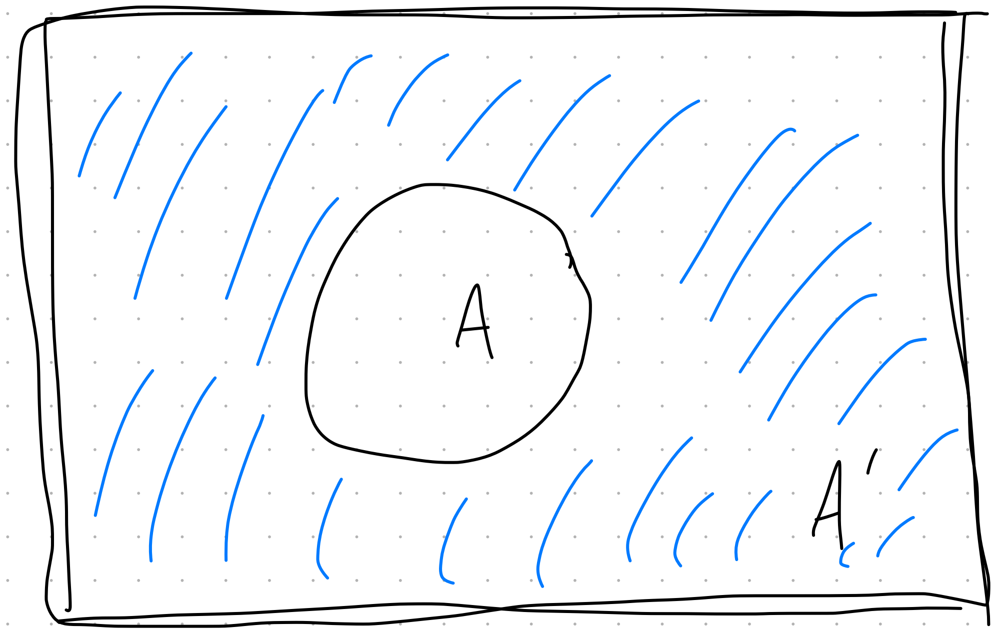
PROBABILITY

EVENT A



$$P(A') = 1 - P(A)$$

$$(A')' = A$$

- $0 \le P(A) \le 1$
- COMPLEMENT

  A', $\bar{A}$, $\neg A$, A^c

- UNION : $\cup$ "OR"
- INTERSECTION : $\cap$ "AND"

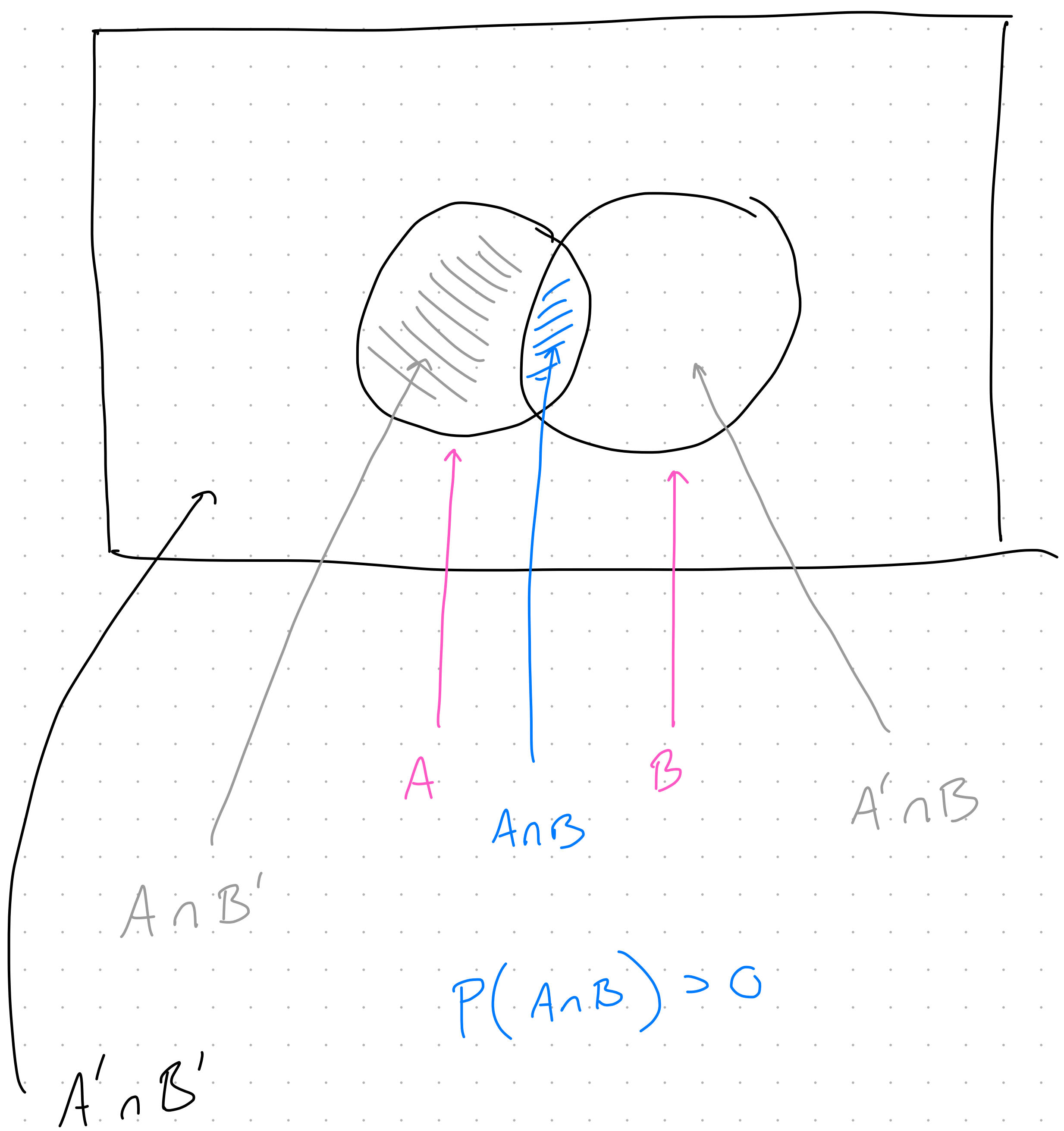
$A \cup A' = \Omega$ "OUTCOME SPACE"

$A \cap A' = \emptyset$ "EMPTY SET"

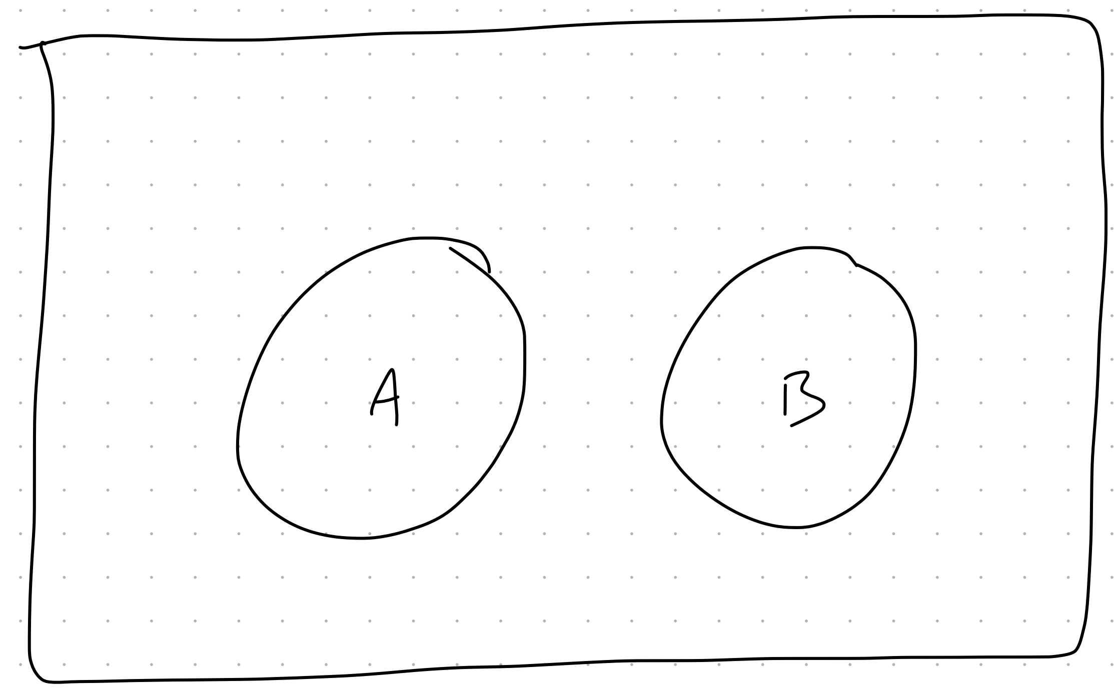
$P(\emptyset) = 0$

$P(\Omega) = 1$

Events A, B



$P(A \cap B) > 0$

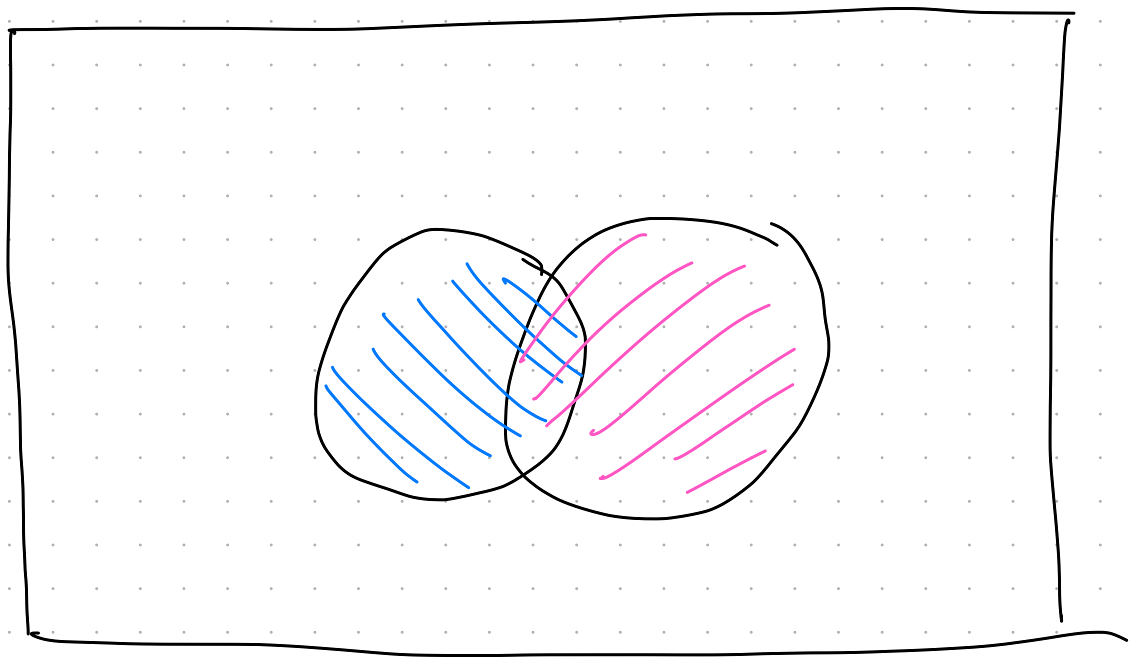


$A \cap B = \phi$

$P(A \cap B) = 0$

A and B are <u>disjoint</u>

Events A, B



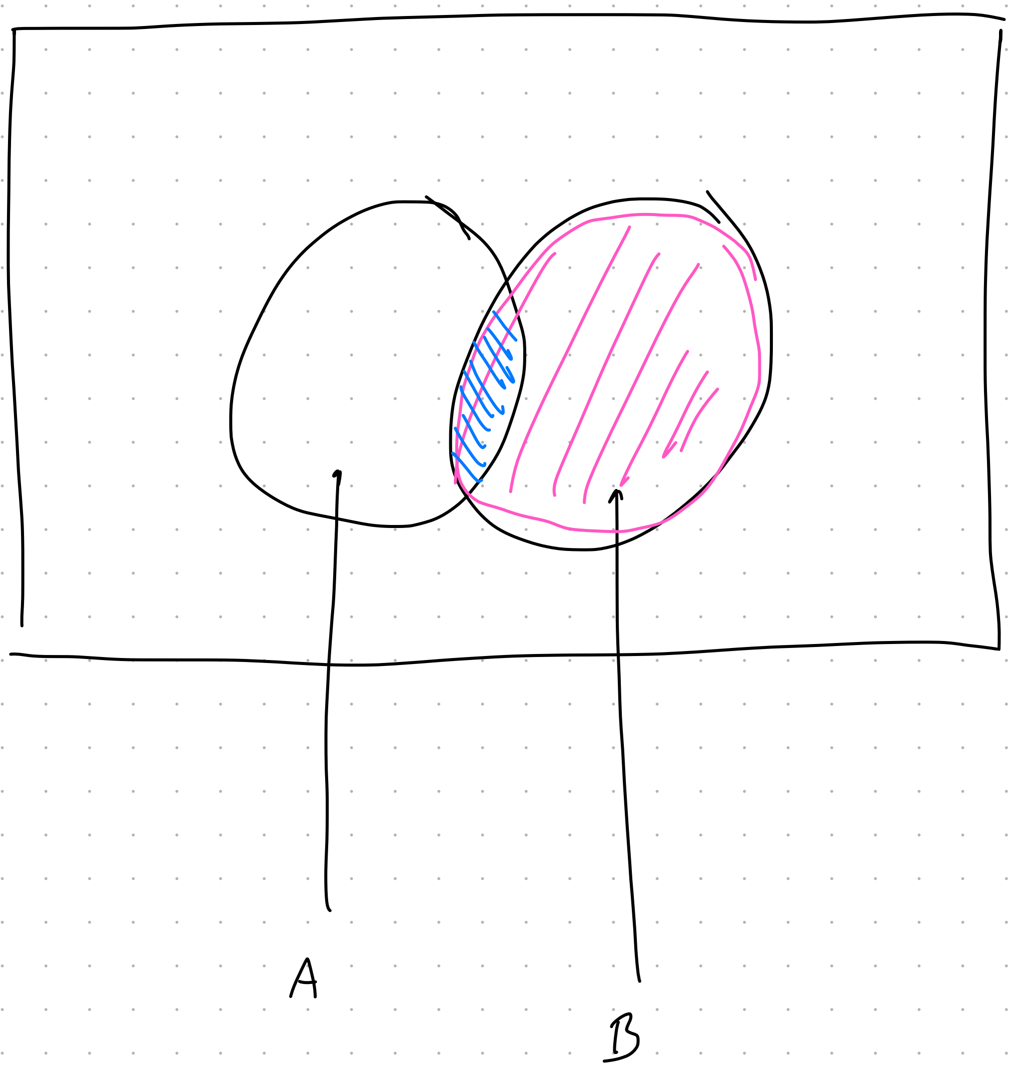
"union"

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

"or"

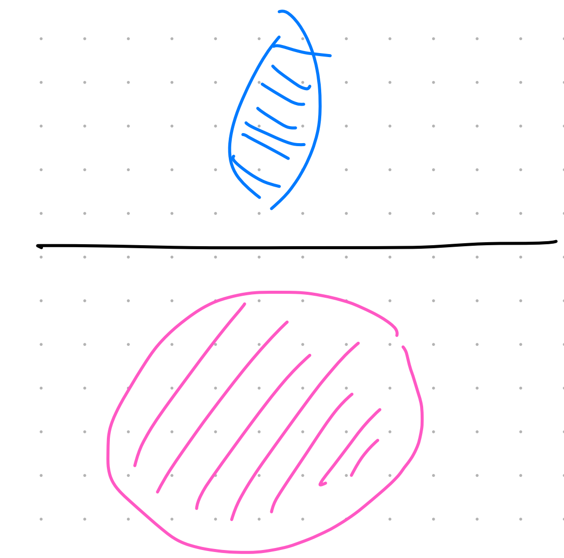
Correction for "over counting"

EVENTS A, B



CONDITIONAL PROBABILITY

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} =$$



"GIVEN"

"CONDITIONING ON B"

EVENTS A, B

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

MULTIPLICATION RULE

$$P(A \cap B) = P(B)\,P(A \mid B) = P(A \mid B)\,P(B) = P(B \mid A)\,P(A)$$

INDEPENDENCE

A, B IND IF

$$P(A \cap B) = P(A)\,P(B)$$

$$P(A \mid B) = P(A)$$

Events A, B

$$P(A') = 1 - P(A)$$

$$P(A' \mid B) = 1 - P(A \mid B)$$

A AND B ARE JUST PLACEHOLDERS

$$P(A') = 1 - P(A)$$

$$P(C') = 1 - P(C)$$

$$P(?') = 1 - P(?)$$

Bayes Theorem

MULTIPLICATION RULE

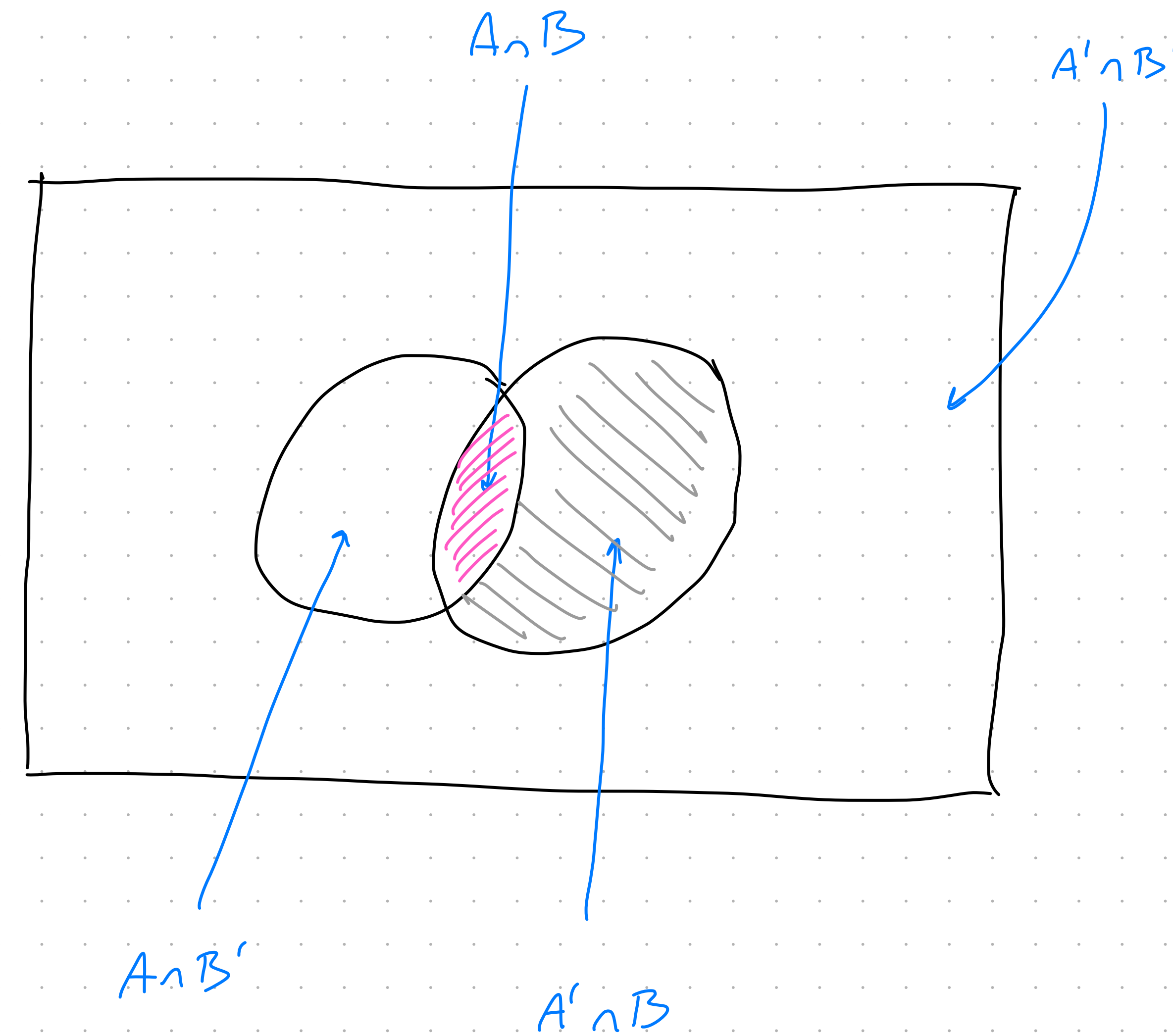
"FLIPPED CONDITIONAL"

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)\,P(B \mid A)}{P(B)}$$

CONDITIONAL PROBABILITY

WHAT IF WE DON'T KNOW THIS?

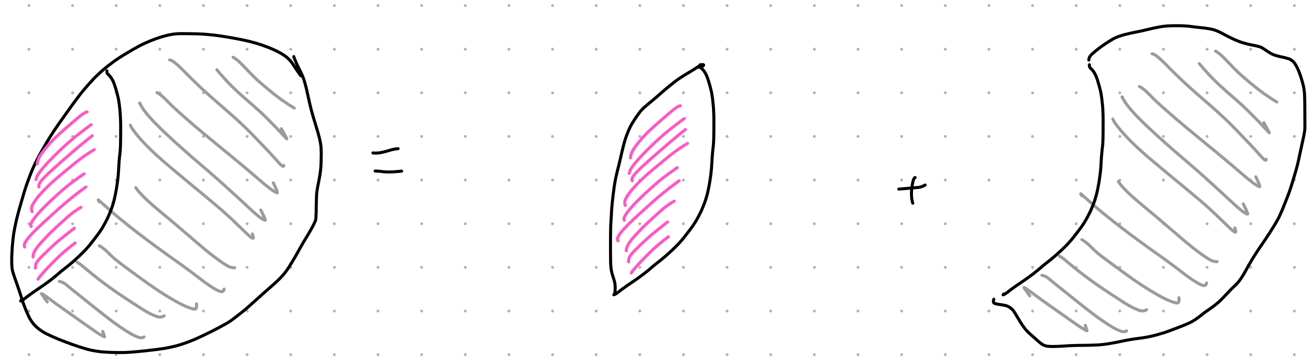
Law of Total Probability



$$P(B) = ?$$

For B to occur, either A or A' must occur!

$$P(B) = P(A \cap B) + P(A' \cap B) = P(A)P(B|A) + P(A')P(B|A')$$

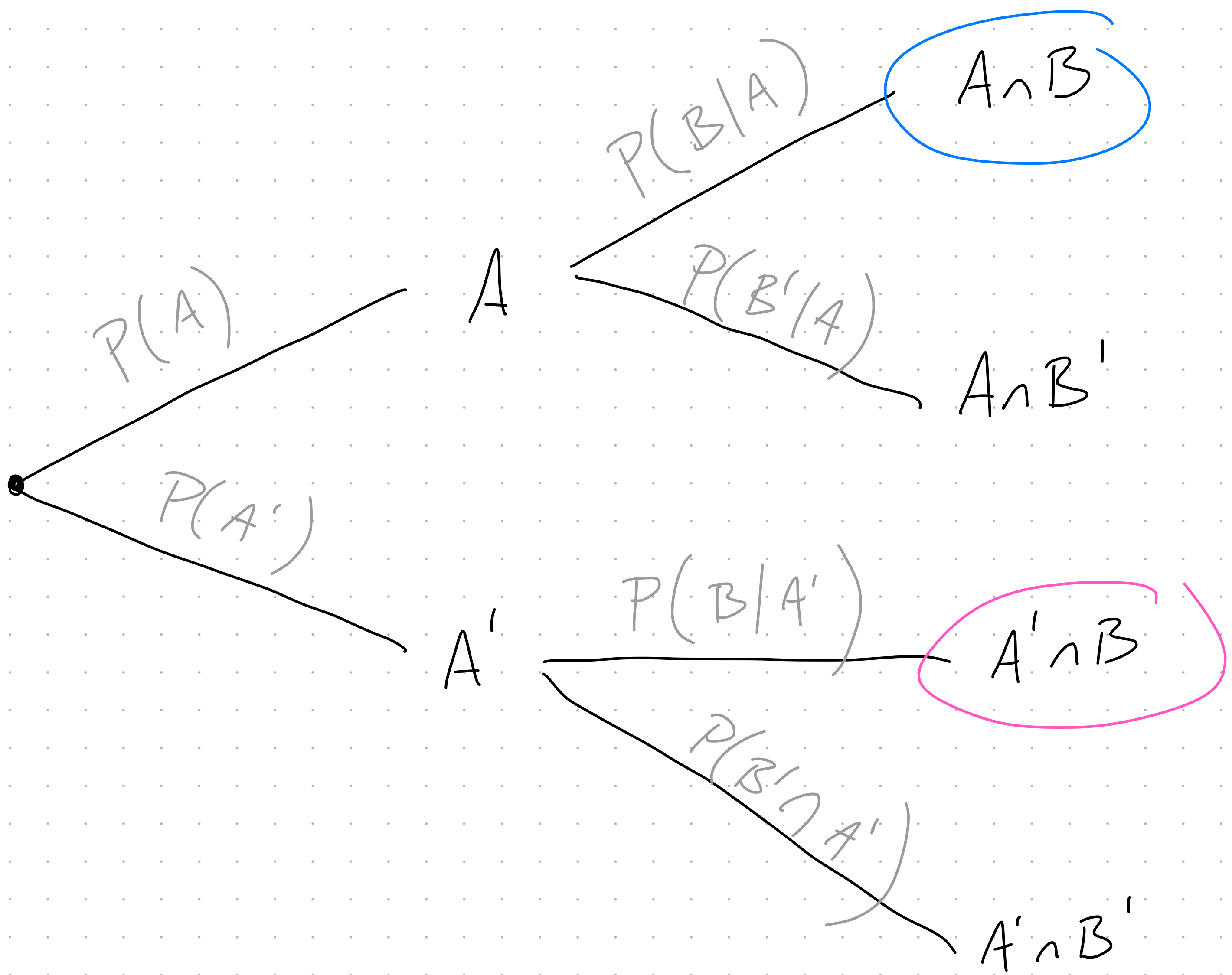


Bayes Theorem

$$\underbrace{P(A|B)}_{\text{"posterior"}} = \frac{P(A \cap B)}{P(B)} = \frac{\underbrace{P(A)}_{\text{"prior"}}\,P(B|A)}{P(B)}$$

$$= \frac{P(A)\,P(B|A)}{P(A)\,P(B|A) + P(A')\,P(B|A')}$$

"TREE VIEW"



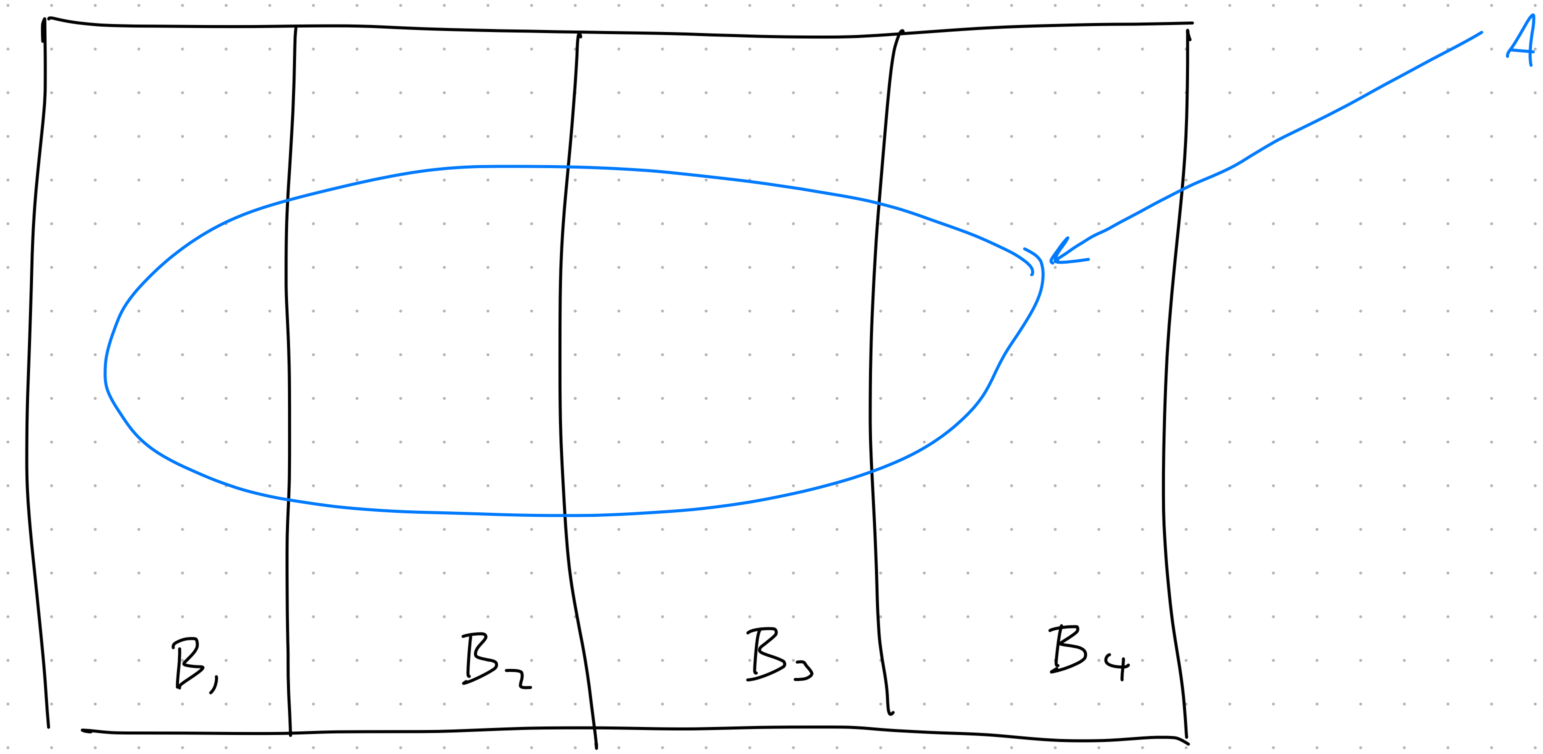
$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P(A \cap B)}{P(A \cap B) + P(A' \cap B)}$$

$$P(A \cap B) = P(A)\, P(B|A)$$

$$P(A' \cap B) = P(A')\, P(B|A')$$

PARTITIONING



$$P(B_4 \mid A) = \frac{P(B_4 \cap A)}{P(A)} = \frac{P(B_4)\,P(A \mid B_4)}{P(A)}$$

$$= \frac{P(B_4)\,P(A \mid B_4)}{\sum_{i=1}^{4} P(B_i)\,P(A \mid B_i)}$$

EXAMPLE / MARGINAL AND JOINT PROBABILITES

RV

	Y=1	Y=2	
X=0	0.1	0.2	0.3
X=2	0.2	0.1	0.3
X=4	0.3	0.1	0.4
	0.6	0.4	

$$P(\underbrace{X=0}_{\text{"A"}} \cap \underbrace{Y=2}_{\text{"B"}}) = 0.2$$

$$P(Y=2 \mid X=4) = \frac{P(Y=2 \cap X=4)}{P(X=4)}$$

Y	P(Y=y)
1	0.6
2	0.4

$$= \frac{0.1}{0.4} = \boxed{0.25}$$