CS 307

Spring 2024

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Decision Trees

Classification

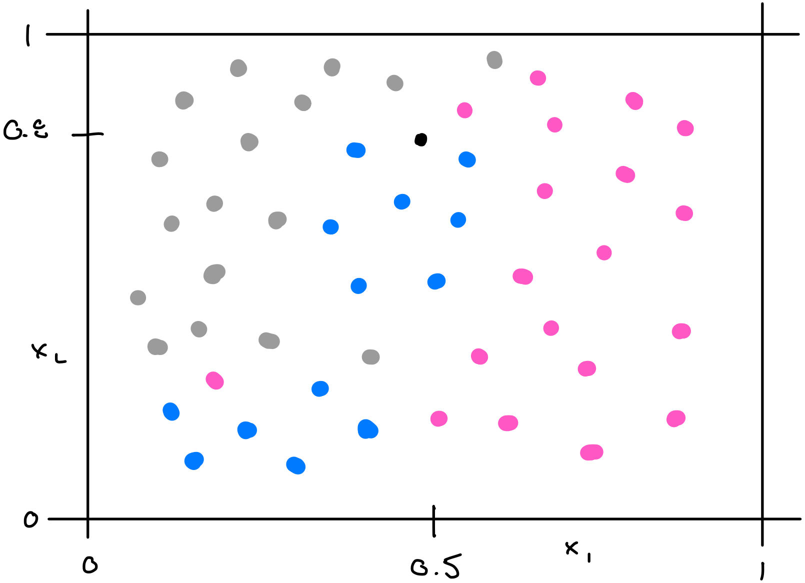
NONPARAMETRIC CLASSIFICATION

ESTIMATING $P[Y = k \mid X = x]$ WITH

- KNN
- TREES

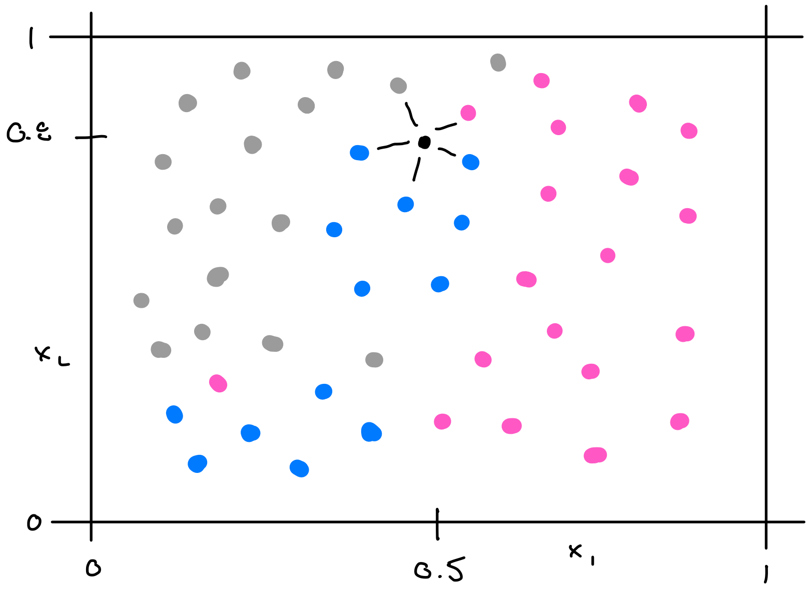
Setup

y	x_1	x_2
A ⋮ A B ⋮ B C ⋮ C	⋮	⋮
?	0.5	0.8



KNN

$$\hat{P}[Y=j \mid X=x] = \frac{1}{k} \sum_{\{i \,:\, x_i \in N_k(x, D)\}} I(y_i = j)$$



WITH $K = 5$, AND $x = (0.5, 0.8)$

$$\hat{P}[Y = A \mid X = x] = 3/5$$

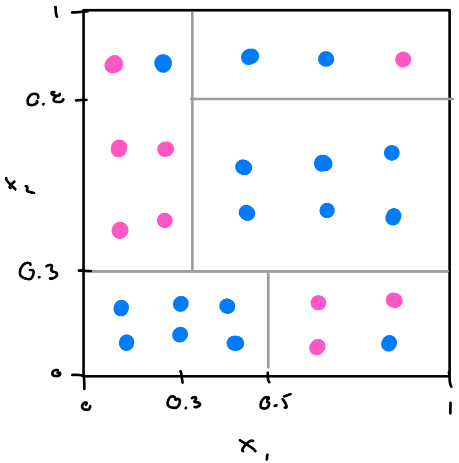
$$\hat{P}[Y = B \mid X = x] = 1/5$$

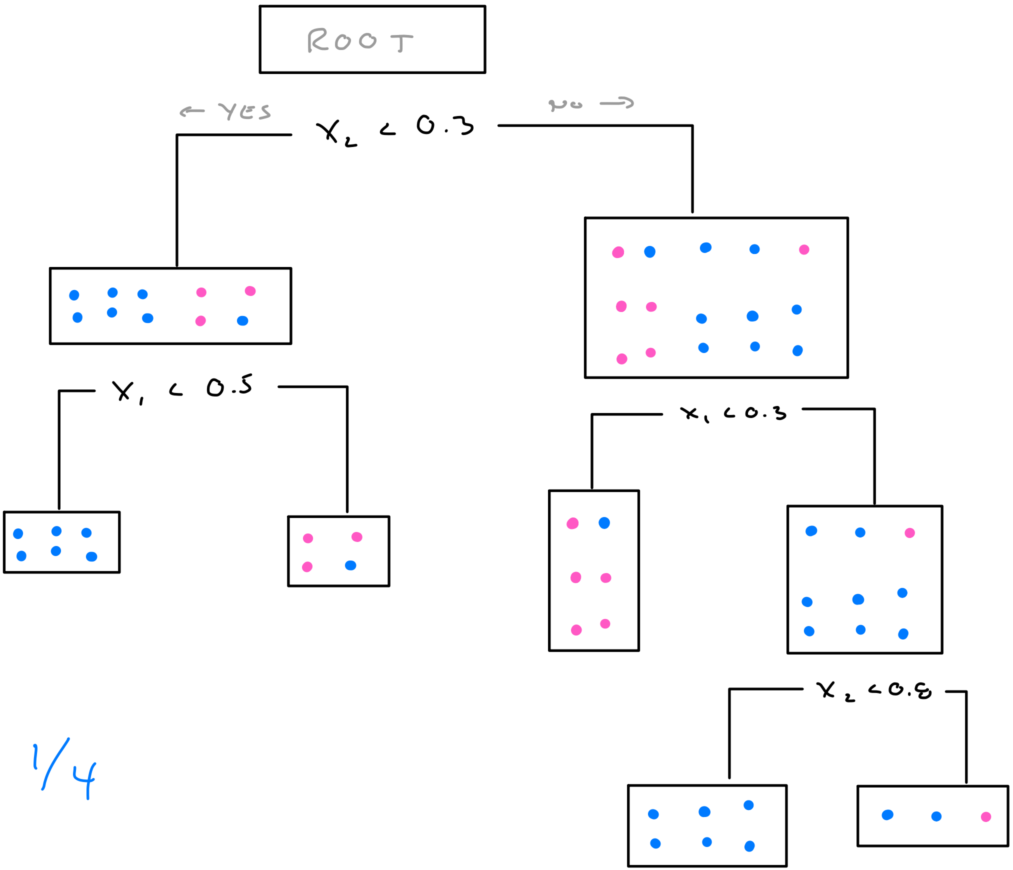
$$\hat{P}[Y = C \mid X = x] = 1/5$$

IF BINARY → USE ODD k

└→ AVOID TIES

Decision Trees





$$\hat{P}\left[Y = \bullet \mid x_1 = 0.9,\ x_2 = 0.1\right] = 1/4$$

Node Probabilities

$$\hat{p}_k = \frac{\sum_i I(y_i = k)\, I(x_i \in A)}{\sum_i I(x_i \in A)}$$

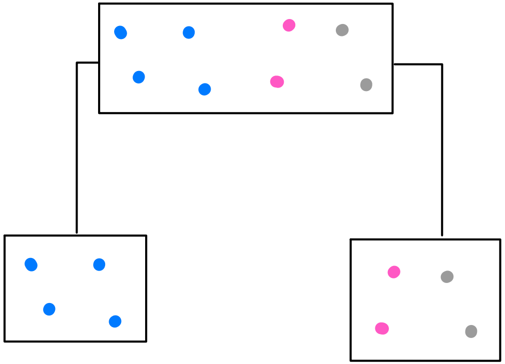
$$\downarrow$$

$$\hat{P}[Y = k \mid x \in N]$$

$\hat{p}_A = 4/8$

$\hat{p}_B = 2/8$

$\hat{p}_C = 2/8$



$\hat{p}_A = 4/4$

$\hat{p}_B = 0$

$\hat{p}_C = 0$

$\hat{p}_A = 0/4$

$\hat{p}_B = 2/4$

$\hat{p}_C = 2/4$

IMPURITY MEASURES FOR CATEGORICAL DATA

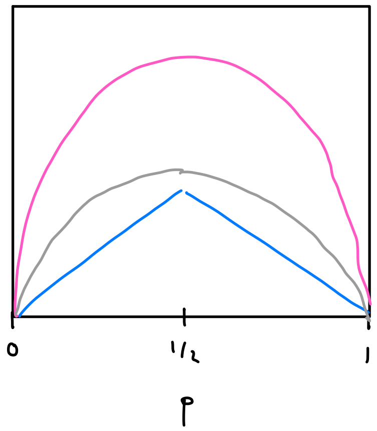
"VARIANCE"

# CATEGORIES

$$\text{GINI}(A) = \sum_{k=1}^{K} \hat{p}_k (1 - \hat{p}_k) = 1 - \sum_{k=1}^{K} \hat{p}_k^2$$

$$\text{ENTROPY}(A) = - \sum_{k=1}^{K} \hat{p}_k \log(\hat{p}_k)$$

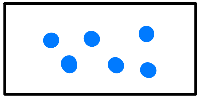
$$\text{ERROR}(A) = 1 - \max_k (\hat{p}_k)$$

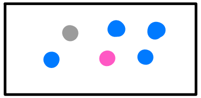


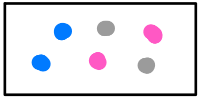
Calculating Gini

$$Gini(A) = \sum_{k=1}^{K} \hat{p}_k (1-\hat{p}_k) = 1 - \sum_{k=1}^{K} \hat{p}_k^2$$

A :







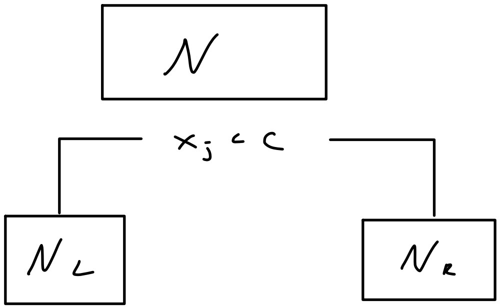
	$\hat{p}_A = 6/6$	$\hat{p}_A = 4/6$	$\hat{p}_A = 2/6$
	$\hat{p}_B = 0/6$	$\hat{p}_B = 1/6$	$\hat{p}_B = 2/6$
	$\hat{p}_C = 0/6$	$\hat{p}_C = 1/6$	$\hat{p}_C = 2/6$
Gini(A)	0	0.5	$0.6\overline{6}$

$$\hookrightarrow = 1 - \left[(4/6)^2 + (1/6)^2 + (1/6)^2 \right]$$

Splitting

Find → feature x_j

Find → cutoff c

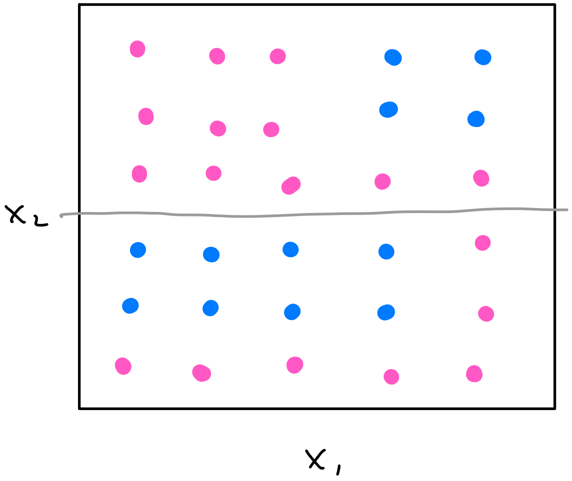


$$\min_{j,c}\left[\frac{|N_L|}{|N|}\,\mathrm{Gini}(N_L)+\frac{|N_R|}{|N|}\,\mathrm{Gini}(N_R)\right]$$

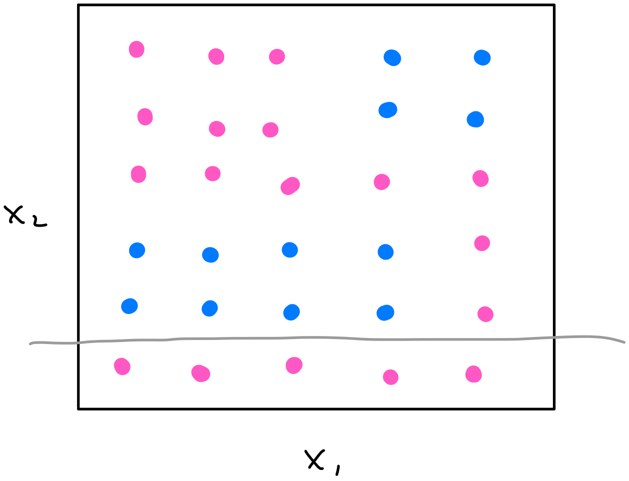
weights

variances

Which Split?



0.44



0.416

```
• FOR EACH CURRENT TERMINAL NODE:
    • IF STOP CONDITIONS MET:
        • SKIP NODE
    • IF NO NODES CAN BE SPLIT:
        • STOP ALGORITHM
    • INIT STORAGE FOR IMPURITIES
    • FOR EACH FEATURE VARIABLE:
        • FOR EACH MIDPOINT, C:
            • CALCULATE AND STORE IMPURITY
    • FIND j AND C THAT GIVE LOWEST IMPURITY
    • SPLIT CURRENT NODE GIVEN J AND C
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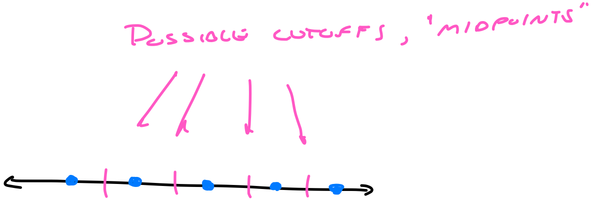
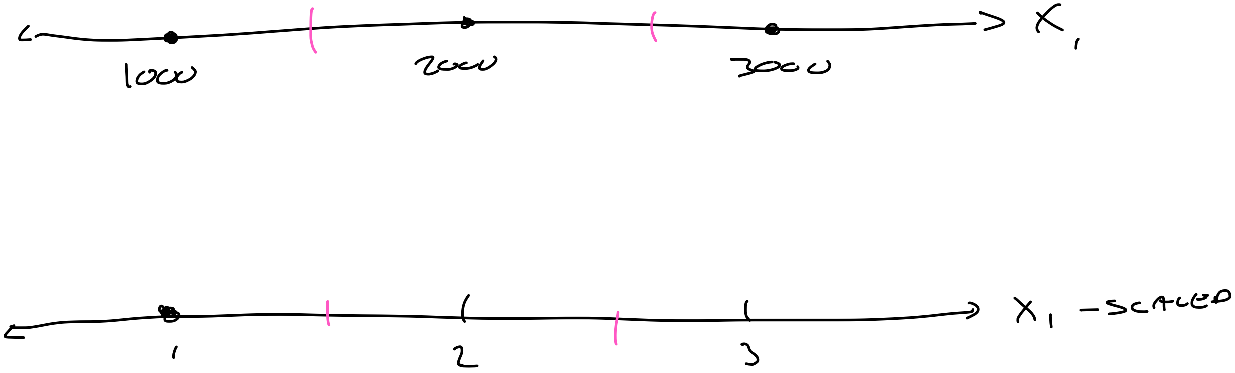


TREES ARE INDIFFERENT TO SCALING


1000
2000
3000
X_1
1
2
3
X_1 -SCALED